

Determination of fragmentation functions and their uncertainties from $e^+ + e^- \rightarrow h + X$ data

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Abstract. Fragmentation functions are determined for pions, kaons, and nucleons by a global analysis of charged-hadron production data in electron-positron annihilation. The optimum functions are obtained in both leading order (LO) and next-to-leading order (NLO) of α_s . It is important that uncertainties of the fragmentation functions are estimated in this work by the Hessian method. We found that the uncertainties are large at small Q^2 and that they are generally reduced in the NLO in comparison with the LO ones. We supply a code for calculating the fragmentation functions and their uncertainties for the pions, kaons, and nucleons at given z and Q^2 .

Keywords: fragmentation function, quark, gluon, hadron production

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INTRODUCTION

Fragmentation functions are used for calculating cross sections of high-energy hadron-production processes. They describe how a hadron is produced in the final state from a parent quark, antiquark, or gluon. Recently, they are becoming increasingly important because semi-inclusive hadron-production processes are investigated for finding the origin of nucleon spin and properties of quark-hadron matters in lepton-nucleon scattering and hadron-hadron collisions.

There have been studies of determining the functions from hadron-production data in electron-positron annihilation [1]. It is known that determined functions are very different between the parametrization groups, for example KKP (Kniehl, Kramer, and Potter) and Kretzer. It is unfortunate that uncertainties of the functions were not estimated although they have been studied in parton distribution functions (PDFs) in the nucleon [2, 3] and nuclei [4]. The major purpose of our work is to investigate the uncertainties of the fragmentation functions [5] in both leading order (LO) and next-to-leading order (NLO) of the running coupling constant α_s .

ANALYSIS METHOD

The fragmentation functions are determined by analyzing the data for charged-hadron production in electron-positron annihilation. The fragmentation function for hadron h is

defined by the cross section for $e^+ + e^- \rightarrow h + X$:

$$F^h(z, Q^2) = \frac{1}{\sigma_{tot}} \frac{d\sigma(e^+e^- \rightarrow hX)}{dz}, \quad (1)$$

where $Q^2 = s$ with the center-of-mass energy \sqrt{s} , and σ_{tot} is the total hadronic cross section. The variable z is given by the hadron energy E_h and the beam energy $\sqrt{s}/2$ by $z \equiv E_h/(\sqrt{s}/2) = 2E_h/Q$. The fragmentation process occurs from primary quarks, anti-quarks, and gluons, so that the function is expressed by their contributions: $F^h(z, Q^2) = \sum_i C_i(z, \alpha_s) \otimes D_i^h(z, Q^2)$, where $D_i^h(z, Q^2)$ is a fragmentation function of the hadron h from a parton i ($= u, d, s, \dots, g$), $C_i(z, \alpha_s)$ is a coefficient function, and \otimes indicates a convolution integral [5].

In our analysis, the functions are defined at fixed Q^2 ($\equiv Q_0^2$) as [5]:

$$D_i^h(z, Q_0^2) = N_i^h z^{\alpha_i^h} (1-z)^{\beta_i^h}, \quad (2)$$

where N_i^h , α_i^h , and β_i^h are parameters. Instead of N_i^h , we use a second moment M_i^h as a parameter in our analysis because its physical meaning is clearer. They are related by $N_i^h = M_i^h / B(\alpha_i^h + 2, \beta_i^h + 1)$, where $B(\alpha_i^h + 2, \beta_i^h + 1)$ is the beta function. The parameters are determined from a χ^2 analysis of the data. The data are taken at large Q^2 , typically $Q^2 = M_Z^2$, and the functions are evolved to experimental Q^2 points by the timelike DGLAP equations. The parameters are determined by minimizing the total χ^2 , $\chi^2 = \sum_j (F_j^{data} - F_j^{theo})^2 / (\sigma_j^{data})^2$, where F_j^{data} and F_j^{theo} are experimental and theoretical fragmentation functions, and σ_j^{data} is an experimental error. The uncertainties of the obtained functions are calculated by the Hessian method [2, 3, 4, 5]:

$$[\delta D_i^h(z)]^2 = \Delta \chi^2 \sum_{j,k} \left(\frac{\partial D_i^h(z, \xi)}{\partial \xi_j} \right)_{\xi} H_{jk}^{-1} \left(\frac{\partial D_i^h(z, \xi)}{\partial \xi_k} \right)_{\xi}, \quad (3)$$

where H_{ij} is the Hessian, ξ_i is a parameter, and the parameter set at the minimum χ^2 is denoted by $\hat{\xi}$.

RESULTS

The parameters are determined by fitting the data for $e^+e^- \rightarrow hX$. As an example, the obtained fragmentation functions in the NLO are compared with experimental data for the charged pions ($\pi^+ + \pi^-$) in Fig. 1, where fractional differences $(Data - Theory)/Theory$ are shown for the fragmentation functions at the experimental Q^2 points. There are fourteen parameters in the pion analysis. The number of the data is 264. The bands indicate the one- σ uncertainty ranges estimated by the Hessian method. The data are generally well explained by our parametrization; however, the DELPHI data significantly deviate from our fit. The total χ^2 is 433.5 in the NLO analysis for the pion so that $\chi^2/\text{d.o.f.}$ is 1.73. In the LO analysis, the total χ^2 is slightly larger.

Determined fragmentation functions for π^+ are shown at $Q^2=1 \text{ GeV}^2$, m_c^2 , m_b^2 , and M_Z^2 in Fig. 2. The functions in the LO and NLO are indicated by the dashed and solid

curves, and the uncertainties in the LO and NLO are shown by the light- and dark-shaded bands, respectively. It is important that the uncertainties are now shown in this work for the fragmentation functions. At small Q^2 ($\sim 1 \text{ GeV}^2$), the uncertainties are large, especially in the LO. The uncertainty bands become smaller in the NLO. It is very difficult to determine the gluon function; however, an improvement can be seen in the NLO because the uncertainty becomes much smaller. The functions are relatively well determined at large Q^2 ($\sim M_Z^2$), where the e^+e^- data are taken.

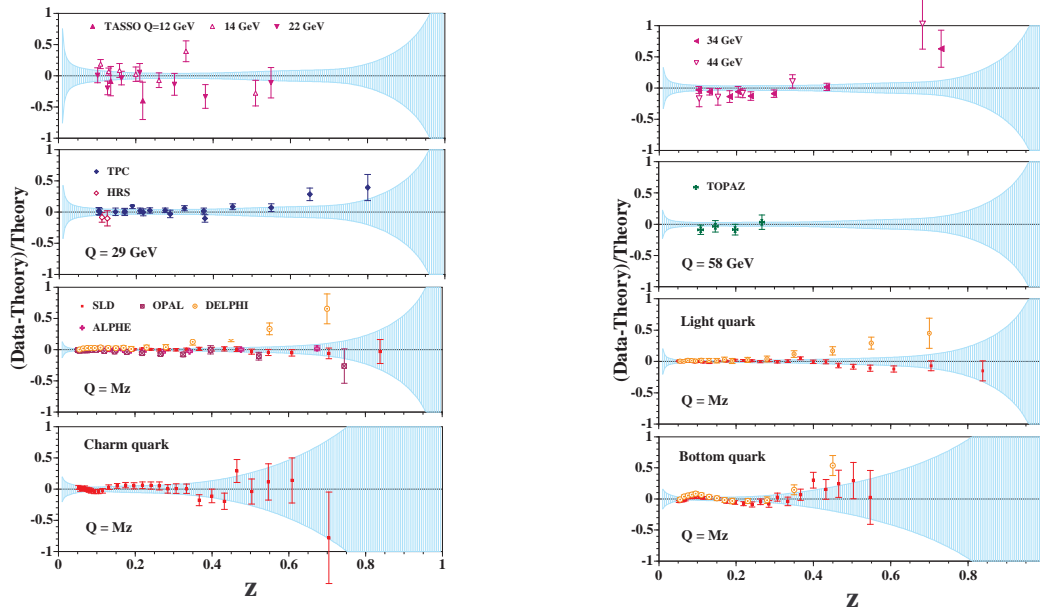


FIGURE 1. Comparison with experimental data for pions [5].

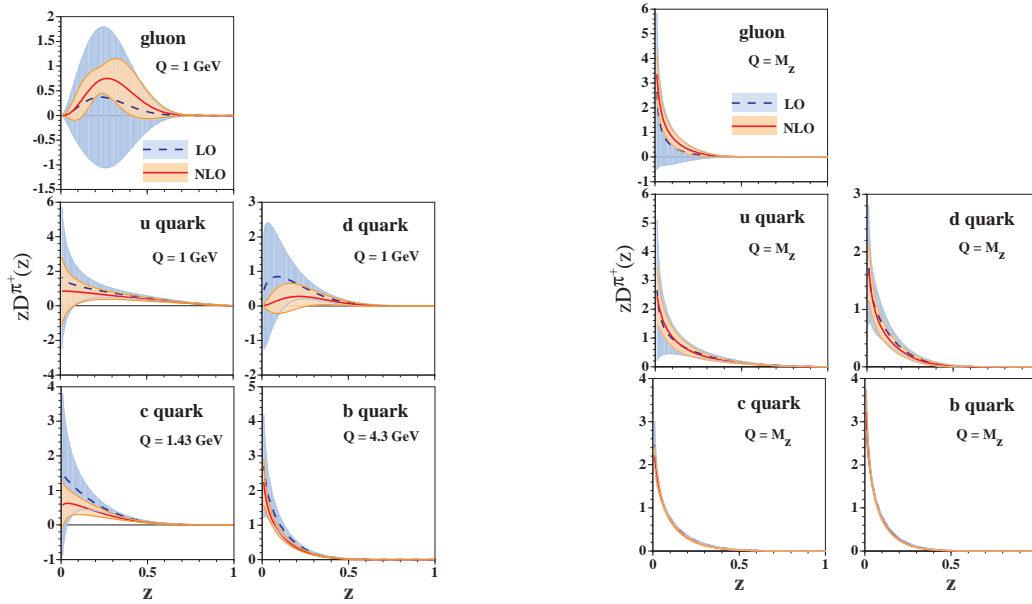


FIGURE 2. Determined fragmentation functions for π^+ at $Q^2 = 1 \text{ GeV}^2$, m_c^2 , m_b^2 , and M_Z^2 [5].

Similar analyses have been done for the kaons and proton/anti-proton. In both cases, the values of $\chi^2/\text{d.o.f.}$ are about two, and the data are generally well explained by our parametrization. As an example, the functions for K^+ are shown in Fig. 3 with their uncertainty bands. The notations are the same as the ones in Fig. 2. The functions have large uncertainty bands also for the kaon at small Q^2 . The large uncertainties suggest that the hadron-production cross sections have large ambiguities if they are calculated in the small p_T region. The obtained functions for the proton as well as the detailed discussions on the pions and kaons are found in Ref. [5].

From the χ^2 analysis of the data for $e^+e^- \rightarrow hX$, the optimum fragmentation functions are determined for the pions, kaons, and nucleons. It is especially important that the uncertainties of the obtained functions are calculated in this work. The results indicate large uncertainties at small Q^2 , which suggests large ambiguities in extracting information, for example on nucleon spin and quark-hadron matters from hadron-production cross sections in lepton-hadron and hadron-hadron reactions. In such studies, it is important to indicate reliable regions of the fragmentation functions by using the uncertainties. We supply our code for calculating not only the optimum fragmentation functions but also their uncertainties at given z and Q^2 for general users [5].

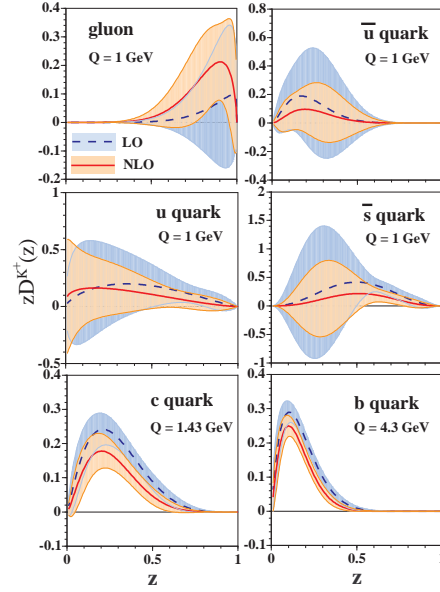


FIGURE 3. Fragmentation functions for K^+ [5].

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